Math 10B with Professor Stankova
Quiz 8; Tuesday, 3/13/2018
Section #203; Time: 9:30 AM
GSI name: Roy Zhao **Name:** ______________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** The covariance can never be negative.

> **Solution:** The covariance of X and $-X$ is $-Var(X)$, which can be negative.

2. True **FALSE** If we get a p value of 0.08 and our significance level is $\alpha = 0.05$, then we accept the null hypothesis.

> **Solution:** We fail to reject the null hypothesis, we can never accept it.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (3 points) You flip a coin 100 times and get 40 heads. What is the 95% confidence interval for what the true probability of getting heads is?

> **Solution:** We want to determine $p = 40/100 = 2/5$ and the standard deviation estimate is $\sqrt{p(1-p)} = \sqrt{3/5(2/5)} = \frac{\sqrt{6}}{5}$. So the 95% confidence interval is $(\mu - 2\sigma/\sqrt{n}, \mu + 2\sigma/\sqrt{n}) = (2/5 - \frac{2\sqrt{6}}{50}, 2/5 + \frac{2\sqrt{6}}{50})$.

(b) (4 points) Is this enough data to say that the coin is biased towards heads with $\alpha = 0.05$? $z(2) = 0.4772$.

> **Solution:** We need to first assume the null hypothesis. This is that the coin is fair and $p = 1/2$. Now when we flip the coin 100 times, we expect a binomial distribution with average $np = 50$ and standard deviation $\sqrt{np(1-p)} = \sqrt{25} = 5$. So the probability of getting at least an extreme scenario as 60 heads is $\frac{1}{2} - z(|40 - 50|/5) = \frac{1}{2} - z(2) = 0.5 - 0.4772 = 0.0228 < \alpha$. Thus we can reject the null hypothesis and say that the coin is biased towards heads.

(c) (3 points) Use the χ^2 test on this coin flip example to determine if the coin is fair. For 1 degree of freedom, the critical value for $\alpha = 0.05$ is 3.841 and for 2 degrees, it is 5.991.

Solution: We got 40 heads and 60 tails and if the coin were fair, we would expect to get 50 heads and 50 tails. Thus our statistic is $\frac{(40-50)^2}{50} + \frac{(60-50)^2}{50} = 4$. Since $4 > 3.841$, we can reject the null hypothesis and say that this coin is not fair.